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1994 J. Phys. A: Math. Gen. 27 L305

(http://iopscience.iop.org/0305-4470/27/10/001)

J. Phys. A: Math. Gen. 27 (1994) L305–L309. Printed in the UK

LETTER TO THE EDITOR

An alternative field-theoretic setting for anyon systems

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Received 21 January 1994, in final form 25 February 1994

Abstract. The cyon model for anyon systems is explicitly analysed using the quantum electrodynamics of particles carrying anomalous magnetic-dipole moments in (2+1) dimensions. Statistical interaction appears together with a further interaction term whose cut-off dependence reflects the unrenormalizable character of the theory.

The by now traditional field-theoretic setting for anyon systems is Chern–Simons (CS) gauge theory [1, 2]. On the other hand, a well known example exhibiting fractional statistics is a cyon, i.e. a composite object consisting of a charged particle bound to a magnetic flux tube [1, 3] or, equivalently, at a large enough space scale, a charged particle in (2+1) dimensions (2D) endowed with a magnetic-dipole moment. This example, although very appealing, is, strangely enough, not used as a working model for 2D systems of identical particles with fractional statistics. This is perhaps due to the feeling that a thorough Lagrangian description for a many-body cyon system interacting with the electromagnetic (EM) field (with Maxwell action) is rather cumbersome.

In this letter, it is shown that the simplicity of the 2D magnetic-dipole interaction Lagrangian allows it to be used effectively. Once the EM potential is integrated out, in the spirit of Feynman's approach to QED [4], a generalization of the 2D Coulomb plus current–current interaction, including statistical interaction, is obtained. Furthermore, the usual Hamiltonian for a system of anyons in terms of integer-statistical particles interacting through fictitious gauge potentials is deduced in the non-relativistic limit. It should be stressed that the procedure is quite distinct from the elimination of CS gauge potentials [1, 2] since the present physical model carries local gauge degrees of freedom.

The motivation is twofold. On one hand, the procedure leads, without using CS terms, to the anyon-model Hamiltonians currently used in several physical applications [5, 6]. On the other hand, this procedure highlights the appearance of the factor two in passing from the cyon magnetic flux to that in the model Hamiltonian. This is not irrelevant as these factors are stressed so often in the literature, e.g. they are stressed at least nine times in [7].

It is worth stressing that the 2D EM potential used in this context should not be confused with the actual EM potential. In fact, 2D electrodynamics is experimentally relevant only in the presence of translational invariance along a given direction. This is not the case, even approximately, in the actual condensed-matter systems to which fractional statistics may, in principle, be relevant.

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To begin with, consider that the interaction energy between particles carrying electric charge e and magnetic-dipole moments μ with the pseudoscalar magnetic field B is given by

$$E_{\rm dip} = -(\mu/e)\int \mathrm{d}^2\boldsymbol{r}\,\rho(\boldsymbol{r})B(\boldsymbol{r}) \tag{1}$$

where ρ denotes the electric-charge surface density.

Contrary to what happens in $(3+1)$ dimensions with the Bargmann–Michel–Telegdi equation [8], the classical theory of particles with spin carrying magnetic-dipole moments in 2D dimensions is quite simple and can be quantized straightforwardly; the expression for the unrenormalizable relativistic interaction Lagrangian, reducing to equation (1) when particles are at rest, is

$$L_{\rm dip} = -(\mu/2ec)\int \mathrm{d}^2\boldsymbol{r}\,\varepsilon^{\nu\rho\tau}j_\nu(\boldsymbol{r})F_{\rho\tau}(\boldsymbol{r}). \tag{2}$$

Here, ε denotes, as usual, the completely antisymmetric tensor density, j^ν is the electric-three-current density $(c\rho,\ j_x,\ j_y)$ and

$$F_{\mu\nu} \equiv A_{\nu,\mu} - A_{\mu,\nu}$$

is the EM-field strength. It should be remarked that, as expected, this interaction breaks chiral invariance when particles with spin are present in 2D [1]. If the Coulomb gauge

$$\nabla\cdot\boldsymbol{A} = \partial_1 A^1 + \partial_2 A^2 = 0$$

is fixed, A_0 can be solved in terms of charge density and the contribution to the dipole-interaction Lagrangian (2) containing

$$A_0(\boldsymbol{r}) = -(1/2\pi)\int \mathrm{d}^2\boldsymbol{r}'\,\rho(\boldsymbol{r}')\ln(|\boldsymbol{r}-\boldsymbol{r}'|)$$

i.e.

$$L'_{\rm dip} = -(\mu/2\pi ec)\int \mathrm{d}^2\boldsymbol{r}\,\boldsymbol{j}(\boldsymbol{r})\cdot\int d^2\boldsymbol{r}'\,\rho(\boldsymbol{r}')\frac{\hat{\boldsymbol{z}}\wedge(\boldsymbol{r}-\boldsymbol{r}')}{|\boldsymbol{r}-\boldsymbol{r}'|^2} \tag{3}$$

gives one half of the expected statistical interaction (once self-interaction is subtracted as usual) and is included in the matter Lagrangian. Following Feynman [4], the effective action is given by

$$S_{\rm eff} = S_{\rm mat} + I \tag{4a}$$

$$\exp(\imath I/\hbar) \equiv N\int D[A_{\rm t}]\exp\left(\frac{\imath}{\hbar}(S_{\rm rad}+S_{\rm dip}+S_{\rm min})\right) \tag{4b}$$

where $A_{\rm t}$ denotes the transverse vector potential (which in 2D has only one mode per wave vector), $D[A_{\rm t}] \equiv D[A]\delta(\nabla\cdot A)$, N is a normalization factor and $S_{\rm mat}$ denotes the matter action.

Here

$$S_{\rm rad} \equiv \tfrac{1}{2}\int \mathrm{d}^2\boldsymbol{r}\,\mathrm{d}t\,\boldsymbol{A}_{\rm t}(\boldsymbol{r},t)(-\partial_t^2/c^2+\nabla^2)\boldsymbol{A}_{\rm t}(\boldsymbol{r},t) \tag{5}$$

denotes the action of the EM radiation

$$S_{\rm dip} \equiv \int \mathrm{d}t\,(L_{\rm dip} - L'_{\rm dip}) \tag{6}$$

and

$$S_{\rm min} \equiv \int (1/c)\boldsymbol{j}(\boldsymbol{r},t)\cdot\boldsymbol{A}_{\rm t}(\boldsymbol{r},t)\,\mathrm{d}^2\boldsymbol{r}\,\mathrm{d}t \tag{7}$$

is the minimal interaction action subtracted from the term giving rise to the Coulomb interaction together with the term originally present in the EM action [4]. This 2D Coulomb interaction is meant to be included in $S_{\rm mat}$ in equation (4a).

The Gaussian functional integration in equation (4) gives $I = \int \mathrm{d}^2\boldsymbol{r}\,\mathrm{d}t\,\mathcal{L}$ with

$$\mathcal{L} = \left[\frac{\nabla}{|\nabla|}\wedge\frac{\boldsymbol{j}}{c} + \frac{\mu}{e}\frac{\Box}{|\nabla|}\rho\right]\frac{\Box_{\rm F}^{-1}}{2}\left[\frac{\nabla}{|\nabla|}\wedge\frac{\boldsymbol{j}}{c} + \frac{\mu}{e}\frac{\Box}{|\nabla|}\rho\right] \tag{8}$$

where $\Box_{\rm F}^{-1} = (-\partial_t^2/c^2+\nabla^2)_{\rm F}^{-1}$ denotes the Feynman propagator (j_0 appears in equation (8) since current conservation was used). Expanding the product in equation (8) gives three terms. The μ-independent term

$$I_{\rm F} = 2\pi\imath\int \mathrm{d}^2\boldsymbol{r}\,\mathrm{d}t\,\mathrm{d}s\,\boldsymbol{j}_t(\boldsymbol{r},t)\frac{\exp[-\imath c|\nabla||t-s|]}{2c|\nabla|}\boldsymbol{j}_{\rm t}(\boldsymbol{r},s) \tag{9}$$

(where $\boldsymbol{j}_{\rm t}$ denotes the purely transverse part of the electric-current density) is the original Feynman expression for the interaction between electric currents. The term in μ^2 can be written as the sum of a (divergent) self-interaction (if hard-core particles are considered as usual when dealing with anyons) to be subtracted just like the Coulomb self-interaction terms and a divergent interaction involving the longitudinal part of the electric current $\boldsymbol{j}_{\rm l}$:

$$I_{\rm div} = \frac{\mu^2}{e^2}\int \mathrm{d}^2\boldsymbol{r}\,\mathrm{d}t\,\rho^2(\boldsymbol{r},t) - \frac{\mu^2}{e^2c^2}\int \mathrm{d}^2\boldsymbol{r}\,\mathrm{d}t\,|\boldsymbol{j}_{\rm l}|^2 \tag{10}$$

This divergence is to be removed by an explicit ultraviolet cut-off Λ as expected from the unrenormalizability of the magnetic-dipole interaction.

From a physical viewpoint, this cut-off reminds us that the model considered here is only meaningful at length scales larger than the single cyon (or equivalently larger than the quasiparticle excitations currently described in terms of anyons). Then, reasonable values for Λ^{-1} are of the order of the linear dimensions of the considered extended object. To be specific, the interaction action $I_{\rm long}$ involving longitudinal currents can be written as

$$I_{\rm long} = -\frac{\mu^2}{e^2c^2}\int \mathrm{d}t\int \mathrm{d}^2\boldsymbol{r}_1\,\mathrm{d}^2\boldsymbol{r}_2\int_{|\boldsymbol{k}|<\Lambda}\mathrm{d}^2\boldsymbol{k}\left(\frac{\boldsymbol{k}}{|\boldsymbol{k}|}\cdot\boldsymbol{j}_1\right)\left(\frac{\boldsymbol{k}}{|\boldsymbol{k}|}\cdot\boldsymbol{j}_2\right)\exp(\imath(\boldsymbol{r}_1-\boldsymbol{r}_2)\cdot\boldsymbol{k}) \tag{11a}$$

where $\boldsymbol{j}_i \equiv \boldsymbol{j}(\boldsymbol{r}_i)$.

On one hand, this term is small at low energy for the presence of the $(|j_1||j_2|/c^2)$ factor. On the other hand, its evaluation gives $I_{\mathrm{long}} = \int \mathrm{d}t\, L_{\mathrm{long}}$ with

$$L_{\mathrm{long}} = -\frac{\mu^2}{e^2c^2}\int \mathrm{d}^2r_1\,\mathrm{d}^2r_2\,\frac{|j_1||j_2|}{|r_1-r_2|^2}[\alpha(1-J_0(\Lambda|r_1-r_2|)+\beta\Lambda|r_1-r_2|J_1(\Lambda|r_1-r_2|)] \tag{11b}$$

where J_0, J_1 denote Bessel functions, $\alpha = -2\pi\cos(\Phi_1+\Phi_2)$, $\beta = 2\pi\cos(\Phi_1)\cos(\Phi_2)$, Φ_1 is the angle between (r_1-r_2) and j_1 and Φ_2 is the angle between (r_1-r_2) and j_2.

Since the asymptotic behaviour as $|r_1-r_2| \to \infty$ of the most slowly decreasing part of equation (11*b*) (i.e. the divergent part as $\Lambda \to \infty$) is

$$L_{\mathrm{long}} \sim -\frac{\mu^2}{e^2c^2}\beta\sqrt{2/\pi}\,\Lambda^{1/2}\int \mathrm{d}^2r_1\,\mathrm{d}^2r_2\,|j_1||j_2|\frac{\cos(\Lambda|r_1-r_2|-3\pi/4)}{|r_1-r_2|^{3/2}} \tag{11c}$$

then this interaction can be reasonably neglected when dealing with low energies and densities where this term, for instance, is dominated by the 2D Coulomb interaction (and even by the actual electric interaction in a 2D condensed-matter system).

Finally, the linear term in the magnetic-dipole moment μ reads

$$I_{\mathrm{mag}} = (\mu/e)\int \mathrm{d}^2r\,\mathrm{d}t\,\left(\frac{\nabla}{|\nabla|}\wedge\frac{j}{c}\right)\frac{\rho}{|\nabla|} \tag{12}$$

which, since $\nabla^2 A_0 = -\rho$ gives another contribution similar to that coming from L'_{dip} in equation (3), thus leading to the correct statistical action which is usually obtained by starting with a CS term [1].

If, to be specific, the non-relativistic-matter Lagrangian is used and the current–current interactions are removed, as usual with non-relativistic QM, then the corresponding Hamiltonian is

$$H = \frac{1}{2m}\sum_{j=1}^{N}|p_j-(e/c)a_j(r_1,\ldots,r_N)|^2-(e^2/2)\sum_{i\neq j}\ln(|r_i-r_j|) \tag{13a}$$

where

$$a_k(r_1,\ldots,r_N) = (\mu/\pi ec)\int \mathrm{d}^2r\,\rho(r)\frac{\hat{z}\wedge(r-r_k)}{|r-r_k|^2} \tag{13b}$$

which, when self-interaction is subtracted, becomes

$$a_k(r_1,\ldots,r_N) = (\mu/\pi c)\sum_{i\neq k}\frac{\hat{z}\wedge(r_i-r_k)}{|r_i-r_k|^2}. \tag{13c}$$

If, in particular, in equations (13), the limit $e \to 0$, $\mu \to \infty$ with $e\mu$ held constant is considered, the free-anyon Hamiltonian in the CS gauge is recovered [1,2]. It should be stressed that, from the present viewpoint, the seemingly unpleasant presence of not just one potential but of as many statistical potentials as particles is no more mysterious than the absence of the self-interactions in the Coulombic potential. Parenthetically, the clearest view of the fictitious gauge terms in the Hamiltonian (13), when dealing with gauge freedom, is, in the authors' opinion, obtained by considering them as components of a unique gauge potential in the $2N$-dimensional configuration space [5].

Finally, since the present approach is based on an explicit classical formulation, it is a natural setting in which to analyse, at a pseudoclassical level, the transformation from the CS gauge to the multivalued gauge [1,2]. This should hopefully be realized by extending the approach described in [9] to suitably generalized Grassmann algebras [10].

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